

Script of the Film
FUNDAMENTAL PRINCIPLES OF FLOW

by

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Shell, bomb, smoke

Stationary foil
with aluminum,
dissolve to

Animation, vectors,
dissolve to

Animation, stream-
lines, dissolve to

Stationary foil
with dye streaks

Moving foil with alu-
minum, dissolve to

Animation, stream-
lines and velocity
vector, dissolve to

Animation, stream-
lines and pathline,
dissolve to

Moving foil with dye

When a fluid such as air or water flows, its various parts - unlike those of a moving solid - usually behave so differently that it is difficult to comprehend at a glance the detailed system of motion. The flow can be made visible, however, by smoke, dye, floats, or the aluminum powder scattered on the surface of the water flowing past this foil. One can then indicate by the length and orientation of arrows the magnitude and direction of the velocity vector at various key points. Finally, one can draw through the field a series of streamlines in such a way that each line would be tangent to the velocity vector at every point of its length. In flow of this sort the same pattern of streamlines is directly revealed by streaks of dye.

If, instead of the fluid moving past the stationary foil, the foil moves through otherwise still fluid, a totally different pattern must prevail. Now the streamlines are displaced with the body, and the velocity vector hence varies not only with relative location but also with time. The moving streamlines show only the instantaneous direction of motion at each point, and the pathline of a single particle deviates considerably from each tangent streamline. The streaklines produced by dye released at fixed points, moreover, are distinct from both streamlines and pathlines except at common points of tangency.

The difference between these two contrasting patterns of motion is that the first was steady or invariable with time and the second was unsteady or variable with time. For the particular case of a foil and fluid with a constant relative velocity between them, one pattern can be changed into the other by the principle of relative motion. In other words, the difference is a matter of viewpoint, regardless of whether the fluid is stationary and the model moving, as in a towing tank, or the model is stationary and the fluid moving, as in wind tunnels and water tunnels. In either case the unsteadiness results from relative motion between the model and the point of observation. Whenever possible, it is advisable to change the unsteady flow into a steady one by means of this principle, not only because the patterns of streamlines, pathlines, and streaklines are then identical and do not change with time, but because the mathematical expressions describing them are also much simpler.

Lecture

Equations on board:

$$a_s = \frac{dv_s}{dt} = \frac{\partial v_s}{\partial t} + \frac{1}{2} \frac{\partial v^2}{\partial s}$$

$$a_n = \frac{dv_n}{dt} = \frac{\partial v_n}{\partial t} + \frac{v^2}{r}$$

Local terms covered

In these two equations for the tangential and normal components of acceleration, the first terms on the right measure the part due to the unsteadiness of the flow (the change in velocity with time at each point), called local acceleration, while the second terms measure the part due to the nonuniformity (the change in velocity with position along a streamline), called convective acceleration. If, by the principle of relative motion, a flow can be changed from unsteady to steady, the local terms become equal to zero and need no longer be considered. This is obviously a great simplification. It is possible, however, only if the flow takes place with constant relative velocity and without change in boundary form or orientation.

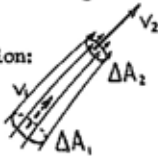
Foil starting from rest and turning

Contraction turning

Waves, first constant then breaking

There are, unfortunately, many cases of flow which cannot be made steady, no matter how one looks at them. For example, a moving body need simply turn or change in speed; a nozzle need merely be varied in discharge or direction; or a wave, instead of progressing through water at constant depth with practically constant speed and form, need only approach and break upon a beach to change its speed and form completely. Such flow can never be made to appear steady.

Animation:



$$\Delta Q = v_1 \Delta A_1 = v_2 \Delta A_2$$

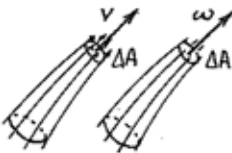
$$v \sim \frac{1}{\Delta A}$$

Flow through contraction, dissolve to

The passage formed by a closed surface controlled in shape by a group of streamlines is known as a stream tube, and the fluid contained within it is called a stream filament. By definition of a streamline, there can be no flow through the walls of a stream tube. Hence, unless the fluid expands or is compressed during motion, at any instant the rate of flow past successive sections of the tube must be the same, in accordance with the principle of continuity. The velocity of the fluid in the filament must therefore vary from section to section in inverse proportion to its cross-sectional area. In two-dimensional or plane motion, the velocity is thus inversely proportional to the spacing of the streamlines. A two-dimensional flow pattern with streamlines enclosing the same incremental rates of flow hence shows at a glance not only the direction but also the relative magnitude of the velocity at all points.

Drawing of streamlines and vectors

Animation:



$$\Delta Q = v_1 \Delta A_1 = v_2 \Delta A_2$$

$$\frac{1}{2} \Delta \Gamma = \omega_1 \Delta A_1 = \omega_2 \Delta A_2$$

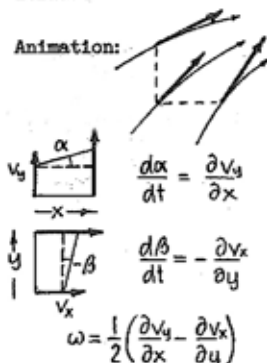
$$v \sim 1/\Delta A \quad \omega \sim 1/\Delta A$$

Spillway vortex

In mechanics it is customary to represent not only linear velocity v but also angular velocity ω vectorially. In fluid motion the angular velocity at a point is called vorticity, and - like streamlines - vortex lines can be imagined tangent to the vectors at all points. The fluid enclosed by a group of vortex lines is called a vortex filament. Just as the product of the velocity and cross-sectional area of a stream filament is known as the rate of flow Q , the product of twice the vorticity and the cross-sectional area of a vortex filament is known as the circulation Γ . A theorem of Helmholtz, like the continuity principle, states that the circulation must be the same at all sections of a vortex filament. The vorticity thus varies inversely with the area of the filament. Such conditions are very graphically illustrated by this vortex forming intermittently behind a model spillway. A less violent

Bridge-pier vortex
Tornado

Animation:



vortex is responsible for eroding the sand bed at the base of a model bridge pier. A very violent kind forms the core of a tornado.

In general, vorticity is not restricted to isolated filaments but varies according to the velocity distribution throughout the moving fluid. It is then necessary to define it mathematically in terms of the velocity gradient. If the component of the velocity vector in one direction varies with distance in a normal direction, then the two ends of a line (or a floating match stick) normal to the velocity components will move at different speeds and the line (or stick) will tend to rotate with an angular speed equal to the velocity gradient. The same is true of a line (or stick) in a direction that is perpendicular to the first, a negative velocity gradient now indicating rotation in the same positive sense. The net vorticity in a given plane depends upon the average of the velocity gradients in the two normal directions. This in turn would be indicated by the average tendency of two lines (or sticks) at right angles to each other to rotate.

View down on rotating tank and aluminum flakes, sticks added

In the case of water in a tank rotating about its axis, the velocity varies directly with the radius. Obviously, lines or sticks either tangential or normal to the streamlines will rotate, just as they would in the vortex filaments seen before. However, if the velocity varies inversely with the radius, as it tends to do in the field of flow surrounding a vortex filament, whether above a drain or in a tornado, a line or stick in the tangential direction will tend to rotate in the opposite sense from one in the normal, but at the same rate. Two perpendicular sticks fastened together will then not rotate at all. Such flow is called irrotational. The central filament itself, of course, is highly rotational.

View down on flow over drain, single stick added off center, cross added off center, cross added at center

Shear flow between parallel walls with cross

Flow in concentric circles is not the only kind that is sometimes highly rotational and sometimes almost irrotational. Uniform flow between parallel walls is usually rotational. On the contrary, either nonuniform or unsteady flow that is rapidly accelerated is very nearly irrotational. Now the important part about irrotational flow is that the flow pattern can be approximated graphically (or mathematically) by the construction of a network of streamlines and normal lines which are so spaced that they divide each other locally into equal parts. As may be shown mathematically, for any shape of the solid boundaries relative to which the fluid moves, there is only one correct flow net, and after a little practice (with rather plentiful use of the eraser) its first approximation can be very rapidly sketched. It is of interest to note that the percolation of either a liquid or a gas through a permeable material (like the earth below a dam) will also be indicated by a flow net if the material is of the same porosity in all directions at all points. Moreover, the same net will correspond

Pier with cross, dissolve to

Pier and flow net

Sketching of another net

Seepage net, dissolve to

Electrical analogy
of same case

to the flow of an electrical current through a conductor - the voltage varying at a maximum rate along a streamline but remaining constant along a normal line.

Lecture

The acceleration of a fluid, like that of any substance possessing mass, requires the action of a force. The types of force which must be considered are three in number: that which is normal to a surface, that which is tangential to a surface, and that which is exerted upon the fluid body as a whole. The body type of force includes not only mass attraction, one result of which is fluid weight, but also inertial effects if the d'Alembert concept of "reversed effective force" is adopted - which we shall not do. Similarly, tangential force, or shear, results from the fluid viscosity. As a matter of fact, if properties such as weight and viscosity are ignored for the present, then the only type of accelerative force left to consider is the normal one, which we call pressure.

Acceleration of
liquid in tube
(slow motion)

Pressure cannot in itself cause acceleration if it is the same on all sides of a fluid element. Only if the pressure is higher at one point than at another will the fluid accelerate in the direction of the pressure drop. Indeed, the force per unit volume in a given direction is equal to the rate at which the pressure per unit area decreases in that direction. According to the Newtonian relationship, this must equal the mass per unit volume - or density - times the corresponding component of the acceleration.

Animation:

$$f_x = -\frac{\partial p}{\partial x} = \rho a_x$$

$$-\frac{\partial p}{\partial s} = \rho \frac{\partial v_s}{\partial t} + \frac{\rho}{2} \frac{\partial v^2}{\partial s}$$

$$-\frac{\partial p}{\partial n} = \rho \frac{\partial v_n}{\partial t} + \frac{\rho v^2}{2}$$

Channel flow
accelerating

It is thus possible to equate the force per unit volume in the tangential and normal directions to the density times the local acceleration and the convective acceleration in those directions. Considering these equations term by term, we see that in unsteady, uniform motion the pressure must decrease tangentially if the magnitude of the fluid velocity is to increase locally with time, as in this uniform passage. It must decrease normally if the direction of the velocity is to change locally with time, as when one plays a hose back and forth in sprinkling a lawn. Likewise, in steady, nonuniform flow a tangential pressure decrease will accompany an increase in velocity along each streamline. And a normal pressure decrease will accompany streamline curvature in that direction.

Channel turning

Convergent section

Bend

Animation:

$$\frac{\rho v^2}{2} + p = f(t)$$

Dissolve to

$$\frac{\rho v^2}{2} + p = C$$

$$\frac{\Delta p}{\rho v_0^2 / 2} = 1 - \left(\frac{v}{v_0}\right)^2$$

We shall discuss a few cases of unsteady flow at a later time. For the present let us note that for steady, irrotational flow the equations of acceleration can be integrated to yield a very convenient and significant pressure-velocity relationship. This states that at any instant the sum of what are sometimes called dynamic pressure and static pressure must be independent of location but not of time. This sum will be independent of time as well if the overall pressure load does not vary. Then, if v_0 be the velocity at some reference point, the pressure-velocity relationship can be given a dimensionless form, the right side of which - and hence also the left (now a sort of local Euler number) - is determined by the flow net.

Streamlines and pressure diagram for cylinder with hemispherical nose

Pitot tube, close-up and in air tunnel with gages

Pitot cylinder

Five-hole tube

This equation serves as the basis for many instruments which are used to measure velocity, pressure, and rate of flow. For example, it states that as the velocity becomes equal to zero at the nose of a body, the pressure there will rise by an amount equal to the dynamic pressure, as in the ordinary stagnation tube. The pressure either partway around the nose or far back along the side of such a tube, on the other hand, is that of the undisturbed flow, and the difference between the tip and side readings is thus the dynamic pressure, as given by this Prandtl-type Pitot tube. Such a Pitot tube is here being used to indicate the velocity distribution alongside a body of revolution; note the different indications of velocity change as the tube leaves the boundary of the body and as it reaches the edge of the stream. Velocity tubes can also have many other forms, such as this two-hole magnitude and direction indicator for two-dimensional flow, or this five-hole instrument for determining the velocity magnitude and direction in general three-dimensional flow.

Air tunnel and pressure gage

Venturi meter and gages

Elbow meter and gages

Instead of the acceleration of flow around a body, the acceleration of flow through a conduit transition can be used in the same manner to produce a pressure change that varies in a known way with the velocity change - for example, the contraction preceding the test section of an air or water tunnel. A similar condition forms the basis of what is known as a Venturi meter. In fact, any boundary form that produces a change in the direction or the magnitude of the velocity - even a pipe elbow - can be used to measure the velocity or rate of flow. Often, of course, as in the case of the elbow, the pressure-velocity relationship is more complex than that for irrotational flow, and the meter must then be calibrated.

Separation from inclined foil

Flow net for abrupt constriction

Dissolve to separation at constriction

Dissolve to water jet beyond constriction

Dissolve to flow net for jet

One reason for the failure of a fluid to move according to the flow net is the phenomenon known as separation, which occurs in boundary regions of too-rapid deceleration. As we shall see in a subsequent film, such separation is often the result of appreciable rotationality (due to viscous shear) along even a smoothly curved boundary. A type of separation which is of interest now is that due to boundary angularity. The flow net for a sharp boundary angle would call for an infinite velocity at the corner, which is physically impossible. Instead, the flow separates from the boundary and the mean surface of separation tends to assume a curvature which is compatible with the requirement of constant pressure along it. The formation of secondary eddies in the separation zone is eliminated in the case of a liquid jet in contact with the atmosphere; the shape of such a jet beyond the point of separation from the boundary can now be predicted from an analysis related to the flow net, and orifices and nozzles thus become useful flow meters once the degree of jet contraction (and thence the Euler number) has been determined.

Another phenomenon that limits use of the pressure-velocity relationship is called cavitation. As Osborne Reynolds demonstrated

Glass Venturi tube with cavitation

Cavitation pocket behind cylinder

Lecture

Drawing on board:



Equations on board:

$$Q = AV$$

$$\frac{\rho V^2}{2} + P = C$$

$$\Sigma F_x = Q \rho \Delta V_x$$

Hose and nozzle, flow starting

Deflection of jet by dynamometer

Hose, nozzle, and gage, joint bursts

Lawn sprinkler

Penstock anchors

Runaway balloon

Passenger jet

Military jet with contrail

Rocket plane

Rocket

Jet-engine pods

with a glass Venturi tube nearly a century ago, the velocity can be increased to the point that the pressure drops locally to the vapor magnitude; the liquid then begins to boil at normal temperature. Much the same phenomenon is seen to occur behind this cylinder in a water tunnel as the velocity is gradually increased. Once the flow has become discontinuous because of the vapor pockets, neither the continuity equation nor the flow net and pressure-velocity relationship can be applied, until the cavitation has progressed to such a stage that the elongated vapor pocket has become comparable to any other stable surface of separation.

The simple continuity and pressure-velocity relationships have permitted the comparison of conditions between various points of a moving fluid. Equally useful forms are obtained if the elementary equations are integrated not only from section to section of a stream tube but also from one stream tube to the next to encompass a considerable portion of the space through which the fluid moves. The integrated continuity equation has much the same form as before, except that V now represents the average velocity over the area of the entire flow section. The integrated acceleration equation takes either of two forms depending upon the manner of derivation. One - the work-energy form - looks much the same as the pressure-velocity relationship for the flow net but is written now in terms of averages over the cross section. The other - the impulse-momentum form - is basically different in that it is a vector rather than a scalar expression. The three equations thus provide essentially independent tools, which can hence be used together in solving problems.

For example, in efflux from a nozzle, the continuity equation will relate the velocity in the hose and jet to the cross-sectional area and the volume rate of flow; the momentum equation will relate the force of the deflected jet to the mass flow rate and velocity change; and the energy equation will relate the pressure ahead of the nozzle to the density and square of the velocity change. The force required to modify the speed or direction of a flowing fluid has been used in one way or another for many centuries. The common lawn sprinkler is the modern counterpart of an ancient Greek steam toy. To prevent a similar movement of hydroelectric penstocks, the forces at bends must be absorbed by tremendous concrete anchors. If such forces are not statically counterbalanced, as in the sprinkler or this runaway balloon, then a useful means of propulsion is at hand. As a matter of fact, the reaction to jet formation is not only the most popular form of propulsion today, but in simplified terms it is a noteworthy example of the intersignificance of the three flow relationships.

Whereas space rockets, like the runaway balloon, carry their own fluid to eject - terrestrial jets - whether air or water - can take in at the front what they later eject at the rear, in accordance

Split pod, showing proportions

Pod in tunnel, propeller starts, unit deflects

Tunnel flow is started, unit returns to null position

with continuity requirements. In this cutaway model of a simplified jet-engine pod, the propeller operates in a larger section than if it were in the open and hence - as seen from the continuity and energy equations - at lower velocity and higher pressure. For comparable reasons, the high-velocity jet will emerge from the rear of the enclosed unit at the same pressure as the surrounding fluid. However, the unit will be displaced in the opposite direction by the reaction to the force required to change the flow from low to high velocity, in accordance with the momentum equation. In actual operation, the thrust is balanced by the drag exerted by the surrounding flow, as simulated by the experimental stream of this air tunnel. For a full understanding of the latter effect, reference must be made to a subsequent film on form resistance.