Script of the Film

EFFECTS OF FLUID COMPRESSIBILITY

by

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Lecture

$E = \rho \frac{dp}{d\rho}$

Steel
$E \approx 28,000,000$ lb/in.^{2}
Water
$E \approx 320,000$ lb/in.^{2}
Air
$E = 1.4p$

In the last four films we have tacitly assumed that we were dealing with a fluid of zero compressibility. Since compressibility and elasticity are inversely related, considering a fluid to be incompressible is the same as assuming its modulus of elasticity $\rho\ dp/d\rho$ to be infinitely great. Now we know that a substance even as rigid as steel has a finite elastic modulus -- less than thirty million pounds per square inch. The modulus of a liquid is about one-hundredth as great, while that of a gas is very nearly the same as its absolute pressure intensity. In the present film we shall see what effect the actual elasticity, or its reciprocal, the compressibility, of a fluid has upon the patterns of motion that we have studied.

$E = \rho \frac{dp}{d\rho}$

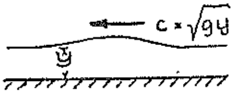
$\frac{\Delta\rho}{\rho} = \frac{\Delta p}{E}$

$c = \sqrt{\frac{E}{\rho}}$

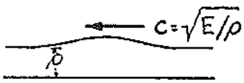
Under static conditions, the effect of compressibility is the variation of the fluid density with the pressure load, as may be seen by writing the definition equation for the elastic modulus in the form of a proportion. Even at the bottom of the ocean, however, the relative increase in the density of the water is only a few percent, because of the great magnitude of the elastic modulus. The density of a gas, on the contrary, can vary from that of its liquid state under very high pressure to its standard value at atmospheric pressure, and we know that the density of air varies from its standard value at the earth's surface to essentially zero in outer space. Under nonstatic conditions, on the other hand, the effect of a local change in pressure is to produce a density change that is propagated elastically from the point of generation with the speed of sound. This speed, which we will call the elastic-wave celerity, depends solely upon the elastic modulus and the density of the fluid under consideration.

Water wave

Animation



As you will recall from the study of elementary physics, there is a great deal of similarity between waves of different types. In fluid motion in particular, a close analogy exists between gravity waves occurring at the surface of a liquid and elastic waves occurring within either liquid or gaseous media. Thus, whereas the schematic diagram of the propagation of a gravity wave represents a simple change in water depth, the same diagram could be used to represent the propagation of an elastic wave, the wave profile now representing a local change in density. Since



elastic waves are usually visible only through use of a rather complex optical system, which we shall illustrate later, this analogy provides a very convenient means of simulating elastic waves by their gravity-wave counterparts.

Successively higher surges

Elastic waves in water really correspond to gravity waves of negligible amplitude, because the change of density of a liquid is always very small compared with the density itself. A gaseous wave, on the other hand, is without limit in amplitude. It has already been seen that a gravity wave of increasing size will become steeper and finally break. The elastic-wave equivalent of such a breaking wave is known as a shock wave, about which more will be said at a later point.

Animation

Valve and record

Valve and cavitation

Change in record with closure speed

As was shown in the first film, the sudden closure of a valve at the end of a long pipe causes an elastic surge to be propagated back and forth between valve and reservoir at sonic speed. At the reservoir end of the pipe it is always reflected negatively, and at the valve end it is always reflected positively, as a result of which the pressure fluctuates rapidly between positive and negative values, the wave form being influenced also by the elasticity and resistance of the pipe. If the initial velocity of the water is very high, the initial pressure rise can be so great as to cause rupture of the pipe. Conversely, the reduction in pressure, when the wave becomes negative, can produce cavitation of the liquid, as is seen here in slow motion, the initial vapor pocket thereafter collapsing against the valve, and then tending to reform repeatedly. Because of the noise that is produced by the sound wave in either case, the phenomenon is very aptly called water hammer. The best way of preventing water hammer is to require that the valves be closed slowly enough to hold the pressure at a safe value. Moreover, pressure tanks containing air set into the line will very effectively cushion the pressure changes.

Waves propagated in flexible tube

Pulse record

Not only is the flexibility of the pipe involved to an appreciable degree, but in the extreme case that of the fluid is negligible in comparison with that of the boundaries through which it flows. This condition is found in the arterial system of the human body. The pulse, which might aptly be called blood hammer, is transmitted from point to point along an artery by virtue of the elasticity of the blood vessel itself. The initial wave is generated, of course, by the pumping action of the heart.

Animation

$$\frac{p}{\rho} = C_1$$

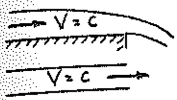
$$V\rho = C_2$$

$$\mathbb{R} = \frac{\rho V D}{\mu} = C_3$$

Overfall scanned

So long as the density changes are small, elastic waves in gases differ little from those in liquids. When the density changes become appreciable, on the other hand, use must be made of the gas equation introduced in the first film. For isothermal flow through a long pipe, the density will vary directly with the pressure and the velocity inversely with the density. Therefore, even though the pressure decreases because of the wall resistance, the Reynolds number and resistance coefficient will remain constant.

Animation



Animation

$$p_1 - p_2 = f\frac{L}{D}\rho\frac{V^2}{2}$$

$$p_1 - p_2 = f\frac{L}{D}\rho\frac{V^2}{2}\left(\frac{2}{2-\Delta\rho/\rho}\right)$$

Such flow is closely analogous to the flow of water in a long channel that ends in an abrupt overfall. As the depth of the flow decreases with distance downstream, the velocity increases, and the Reynolds number remains constant. The limiting depth of the water is that corresponding to critical flow near the brink. Similarly, the limiting density of the gas at the outlet will be that corresponding to the velocity of sound at atmospheric temperature. In flow with constant density the pressure drop varies according to the usual Weisbach equation. If the change in density is appreciable, however, the pressure drop will be increased in accordance with the density change, as indicated by the term within parentheses.

Animation

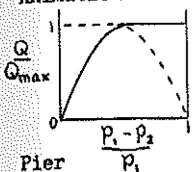
$$\frac{p^{1/k}}{\rho} = C$$

$$p_2 - p_1 = \frac{\rho}{2}\left(V_1^2 - V_2^2\right)$$

$$p_2 - p_1 = \frac{\rho_1}{2}\left(V_1^2 - V_2^2\right)\left[1 + \frac{\rho_1\left(V_1^2 - V_2^2\right)}{4kp_1} \cdots\right.$$

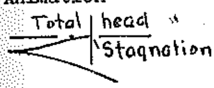
Constriction

Animation



Pier

Animation



If, instead of the temperature, it is the heat content that does not change, the pressure-density equation involves the adiabatic constant k. If this is combined with the equation of motion for negligible resistance, a result will be obtained which differs from that for an incompressible fluid by an amount depending on the density and the change in velocity between any two sections. In other words, in adiabatic flow a pressure change produces a temperature change which augments the change in pressure. Otherwise the flow is closely analogous to that of water in the case of a local constriction. Under normal conditions the flow at the constriction passes from the subcritical state to the critical at the crest as the tailwater is lowered. This corresponds to conditions of maximum discharge, which remains constant as the tailwater continues to fall. Similarly, the discharge through a gas nozzle becomes a maximum when the pressure reaches its critical value, and thereafter the velocity remains that of sound, corresponding to the horizontal solid line at the upper right, even if the external pressure continues to drop, according to the sloping broken line at the lower right. Adiabatic flow can involve standing waves just like the gravity flow of water, the flow at each section being equal to the celerity of either an elastic or a gravity wave under the corresponding conditions. If free-surface flow approaches a zone of stagnation, say on the front face of a pier, the surface will tend to rise in a standing wave to a level coinciding with the line of total head. Much the same thing occurs when a gas approaches a state of stagnation, say at the nose of a pier or Pitot tube, but the temperature rise now causes the pressure rise to be still greater.

Wavelets

$\frac{V}{c} \approx 0$

$\frac{V}{c} \approx 0.6$

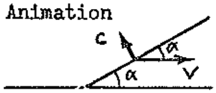
$\frac{V}{c} \approx 1$

$\frac{V}{c} \approx 1.2$

The two-dimensional aspects of wave propagation are even more significant than the one-dimensional. If a periodic disturbance exists in a fluid moving at a very small velocity compared with the wave celerity, the series of wavelets that are formed will be practically symmetrical about their source. As the velocity of the flow increases, however, the wavelets will move upstream less rapidly and downstream more rapidly, with the result that the pattern will become more and more asymmetric. When the velocity of the fluid becomes equal to the celerity, the upstream motion will be reduced to zero and the wavelets will all be tangent to

$\frac{V}{c} \approx 2$

Animation



$\sin \alpha = \frac{c}{V} = \frac{1}{\mathbb{F}}$ or $\frac{1}{\mathbb{M}}$

a normal line passing through the source. Still further increase in the velocity of the flow will cause all portions of the wavelets to be carried downstream, the lines of tangency subtending an angle which becomes smaller and smaller than 180° as the ratio of fluid velocity to wave celerity increases above unity. Half this angle is known as the Mach angle. The reciprocal of its sine is simply the Froude number, in the case of gravity waves. In the case of elastic waves, its counterpart is known as the Mach number.

Curved wall

Angular wall

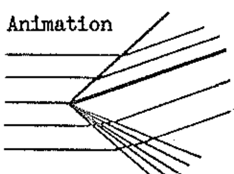
Any body that is involved in relative motion with a fluid at a velocity that is either a fraction or a multiple of the wave celerity will produce a comparable flow pattern. Each element of a curved wall can be regarded as a disturbance producing its own wavelets, the converging wavelets combining to form a surge or shock wave which eventually moves upstream as the angle, and hence the wave, becomes too great. If the change in wall direction is abrupt rather than gradual, the shock wave will begin right at the discontinuity, again starting to move upstream as the angle of the wall becomes excessive.

Sphere

As shown in these sequences made at the Aberdeen Proving Ground, the optical method known as Schlieren produces a diffraction of light that varies with the density or pressure of a flowing gas. Here subcritical flow is developing past a sphere, the zone of positive pressure above the axis becoming green and that of negative pressure red. Below the axis the opposite is true. As the flow becomes supercritical, the shock wave that forms is seen as green above, and the zone of rarefaction just behind as red. Note that the shock wave is well ahead of the boundary at the line of symmetry. This distance decreases as the Mach number increases, the latter change being indicated by a decrease in the Mach angle. The higher the Mach number, of course, the greater the pressure within the shock wave, as indicated by the relative intensity of the color.

Plate

Animation



Previous sequences of water waves at a change in wall alignment are reproduced by the shock wave on the upstream side of this inclined plate. On the downstream side, however, there is no shock wave but a zone of rarefaction, which is represented schematically by a series of divergent negative wavelets. As the streamlines pass through the shock wave, they necessarily come closer together, and the pressure rises. On passing through the divergent wavelets, on the contrary, the streamlines move farther apart, and the pressure drops. Such changes characterize all of the more complex phenomena of supersonic flow.

Blunt nose

Here a blunt-nosed cylinder, for example, is first seen in subsonic flow. Above the axis, the zone of positive pressure is green, as before, and the separation zone of negative pressure is red; below the axis the opposite is again true. The same color differentiation also applies to the supersonic flow that is just developing. In terms of the analogy between gravity waves and elastic waves, this now corresponds to a barge or scow traveling faster than the wave that it produces. As with the sphere seen earlier, the shock wave is formed some distance ahead of the nose, this distance decreasing as the Mach number increases. Evidently, whereas bodies for subsonic flow are streamlined at the rear, those for supersonic flow should be pointed at the nose, and the faster they are intended to go, compared to the wave celerity, the sharper the nose should be. As may be noted from this series of nose angles at the same Mach number, the sharper the point, the smaller the shock wave, and the lower the pressure buildup that results.

90° nose

45° nose

The same principle governs the form of lifting vanes. The subsonic profile is more or less rounded at the leading edge, which produces an appreciable shock wave when the flow is supersonic. Making the leading edge sharp is seen to reduce the intensity of the shock, and hence the pressure rise and drag, to a considerable degree.

Subsonic foil

Supersonic foil

A supersonic plane must be so designed that the shock waves produced by fuselage and wings are of minimum intensity, since they give rise not only to the drag of the plane but also to the disturbance which reaches listeners as a sonic boom when the shock wave passes them. The effect of the swept-back wings is clearly seen as the plane model is rotated about its axis of symmetry. Every shock wave that the Schlieren process makes visible is the source of both drag and noise, and everything possible must be done to reduce the wave magnitude and thus increase the efficiency of supersonic flight.

Plane silhouette