Script of the Film

FLUID MOTION IN A GRAVITATIONAL FIELD

by

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Satellite launching

Man undergoing
   acceleration

Hand releases ball,
   which falls

Ball does not fall

Man apparently
   weightless in space

Man blows bubbles

Animation

$$\left(f_p\right)_s = -\frac{\partial p}{\partial s}$$

$$\left(f_g\right)_s = -\gamma\frac{\partial z}{\partial s}$$

$$f_s = -\frac{\partial}{\partial s}\left(p+\gamma z\right) = \rho a_s$$

$$-\frac{\partial}{\partial s}\left(\frac{p}{\gamma}+z\right) = \frac{a_s}{g}$$

$$-\frac{\partial}{\partial s}\left(\frac{p}{\gamma}+z\right) = 0$$

$$\frac{p}{\gamma}+z = C$$

Rotating tank,
   dissolve to

Vector diagram and
   lines of constant
   p and h

Animation

$$\frac{\Delta h}{V_0^2/2g} \quad \left(\frac{V}{V_0}\right)^2 \quad V_0$$

   dissolve to

Actual passage
   with manometers

Passage inclines,
   heads do not change

Gravity has been brought to popular attention in recent years through the extremes of travel into space. For instance, the forces involved in the acceleration and deceleration of satellites - simulated in this centrifugal trainer - are measured in g's, or multiples of the mass attraction exerted by the earth. Now the effects of gravity, or weight, are several. As an unbalanced force it will produce acceleration. As a balanced force it will only produce stress in the object upon which it acts. Except for such stress, the object might then just as well be weightless, like bodies in space. Quite similar to the zero gravity of outer space is the neutral buoyancy experienced when swimming under water.

If we combine the expression for the force per unit volume due to pressure gradient with that for the force per unit volume due to gravity - which involves the specific weight or fluid weight per unit volume γ - a simple expression for any component of the two together will result. This can then be equated to the product of the density and the corresponding component of acceleration. If it is noted that the ratio of γ to ρ is the gravitational acceleration g, division of all terms of the foregoing equation by γ will show that the sum of pressure head and elevation must change at a rate proportional to the relative acceleration in that direction. In a direction in which there is no acceleration, the sum must be constant. This condition is known as hydrostatic pressure distribution. In a cylindrical tank of water that is rotating with a constant angular speed, for example, the pressure distribution is nonhydrostatic radially because of the centripetal acceleration, but hydrostatic vertically because there is no vertical acceleration.
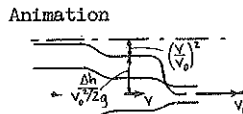
Now if the sum of pressure head and elevation is replaced by the piezometric head h, then the dimensionless pressure-velocity relationship for nongravitational flow will be found to have as its gravitational counterpart an almost identical expression. That is, the pressure-distribution curves for flow without gravitational action are identical to those for the variation of h with flow through the same boundaries in a gravitational field. Since h represents height to which liquid will rise in a glass manometer connected to a piezometric opening, the constancy of the liquid levels in the manometers shows the lack of dependence of the piezometric head upon the orientation of this flow passage. In confined flow, therefore,

$$\frac{\rho v^2}{2} + p = C$$
$$\frac{v^2}{2g} + \frac{p}{\gamma} + z = H$$
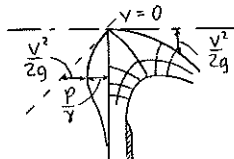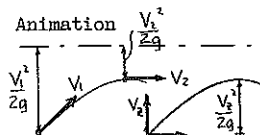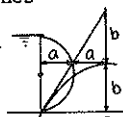
fluid weight has no effect upon the flow pattern. Its sole influence is to increase the pressure head p/γ in proportion to the decrease in elevation.

The dimensional form of the pressure-velocity relationship has its gravitational counterpart in what is known as the Bernoulli equation, which states that the sum of velocity head, pressure head, and elevation must yield the same total head H at all points in steady, irrotational flow. Since the total head is the height to which liquid would rise in a manometer connected to a stagnation tube, the Bernoulli terms are especially suitable to graphical representation, for the velocity head is the distance between the horizontal line of total head and the line of piezometric head below. A gradual change in rate of flow causes the two sets of manometer columns to rise or fall proportionate distances.

Gravity exerts an accelerative effect upon fluid motion only when the flow in question has a locally inclined free surface. This is best illustrated by a liquid flowing in contact with the atmosphere, as in this vertical jet. Since the free surface is a line of zero pressure, the stagnation point coincides with the maximum level attainable - the line of total head - and the velocity head increases on either side of the stagnation point as the surface decreases in elevation. The velocity at points within the flow is related to that of free-surface points by the flow net. The pressure at any internal point then follows from the Bernoulli equation. It is evident that an internal pressure must exist, for only the lateral pressure gradient is available to produce the required horizontal acceleration of each half of the flow.

If the pressure is zero not only along the free surface but throughout the moving liquid, as in this jet, then conditions correspond to those of free fall. In the light of the Bernoulli equation, the sum of the velocity head and the centerline elevation of a liquid jet must be constant. In the light of the equations of physics, the horizontal velocity component must be constant, since there is no horizontal acceleration, and the square of the vertical component alone hence varies inversely with the elevation. Torricelli, a pupil of Galileo, reasoned from his master's laws of free fall that a vertical jet should rise practically as high as the free surface in the supply tank. Moreover, for an inclined jet, from the geometry of the parabola and the circle, he showed that the intercept of the initial tangent with a circle based at the point of tangency determines the location of the vertex - and hence all other points - of the parabolic jet profile. Evidently, the maximum length of trajectory is attained at 45°, when horizontal and vertical components are initially equal. It follows that jets from orifices in the side of a tank equidistant from top and bottom will intersect the bottom line at the same point, and hence that an orifice at midheight will

Tank with equidistant
orifices

Fire stream
diffusing

Jet from second film

Jet from slot at moder-
ate head, total-head
line superposed

Dissolve on $F = \dfrac{v}{\sqrt{gb}}$

Jets at lower
total heads

Flow over bottom
edge of slot

Sharp-crested
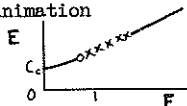ventilated weir

Ventilation stopped,
successive stages
of nappe

Ventilated nappe

dissolve to

Spillway

Low weir, gradually
submerged

Same weir, overshot
by flow from
sluice gate

Animation

Drowning of gate

Animation

produce a jet with the greatest horizontal throw. High-velocity
jets from fire nozzles fail to duplicate these parabolic trajecto-
ries because the intense turbulence generated in the hose causes
the flow to break into spray. Further effects of turbulence will
be illustrated in the next film of this series.

A jet shown in the second film to illustrate the free-streamline
principle was photographed in such a way as to eliminate the effect
of gravity. If a similar jet is now observed in a gravitational
field, the relative influence of the fluid weight will become the
greater as the velocity head relative to the orifice dimension de-
creases. The square root of twice this ratio is the Froude number
introduced in the first film. Evidently, the lower the line of total
head, or the smaller the Froude number, the greater will be the rela-
tive effect of gravity on the deflection of the free stream.

If the head decreases till the top half of the boundary becomes in-
effective, the lower half will act as a weir, over which the water
will flow with a Froude number that is no longer independently vari-
able but depends upon the geometry of the flow boundaries. If a weir
is used for flow measurement, the lower as well as the upper surface
of the nappe must be kept at atmospheric pressure by ventilation.
The rise in water level under the nappe is now purely a dynamic ef-
fect involved in the deflection of the stream by the floor. If, how-
ever, the ventilation tube is removed, the air under the nappe will
gradually be carried away; as this occurs, the difference in pressure
will cause the nappe to be depressed in accordance with the flow net
and Bernoulli equation. In passing it might be noted that large
spillways are built according to the profile of the under surface of
a ventilated nappe, on the reasonable assumption that the resistance
of the solid boundary will not appreciably change the distribution
of velocity and pressure at the crest.

Although the Froude number of the ventilated weir is fixed by the
geometry, it can be reduced toward zero if the downstream side is
gradually submerged by the tailwater. On the other hand, it can be
increased far above unity if the flow is made to approach the weir
with sufficient velocity to overshoot it, as is seen here at succes-
sively greater values of the Froude number. However, in the plot
of the Euler number or discharge coefficient against the Froude num-
ber, which shows these two distinct zones of operation, there will
be a gap between them in which flow at the assumed depth will be
physically impossible; in other words, as the Froude number is re-
duced toward unity, the approach section will abruptly become sub-
merged by backwater.

Further perspective in the matter is obtained by considering flow
that is nearly enough uniform for the one-dimensional approximation
to apply. The total head relative to the channel bottom is the so-
called specific head or sum of depth and velocity head. For

$$H_0 = \frac{V^2}{2g} + d$$
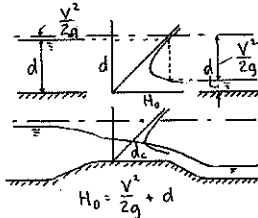
Actual weir, sub-
critical to super-

Sub- to sub-

Super- to sub-

Super- to super-

Backwater

Ship waves

Wave refraction
by island

Breakers at beach

Oscillatory waves

Animation
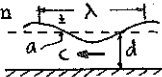
Shallow-water wave,
dissolve on

$$c = \sqrt{gd}\left[1 + f\left(\frac{a}{d}\right)\right]$$

Deep-water wave,
dissolve on

$$c = \sqrt{\frac{g\lambda}{2\pi}}\left[1 + f\left(\frac{a}{\lambda}\right)\right]$$

Breaking waves

Particle paths

Oscillatory waves
and generator

Wall, oscillatory
waves approach
and are reflected

Generator, solitary
wave is produced,
propagates, and is
reflected

constant discharge this sum will vary according to the two-branched curve known as the specific-head diagram, which shows that for a given discharge and specific head two different depths are possible. The flow can pass between them by going over a bottom rise of just the right height to produce the minimum specific head or critical depth at its crest. Critical conditions correspond to a Froude number of unity. Flows with subcritical velocities (depths greater than the critical) thus correspond to Froude numbers less than unity, and vice versa. The flow downstream can hence be either supercritical or subcritical without affecting that upstream. Evidently, it is also possible to pass from the supercritical to the subcritical, at that particular discharge producing critical flow at the crest, or once again to the supercritical on the downstream side. As in the case of the sharp-crested weir, moreover, there is an intermediate range in which flow is physically impossible; thus, if the flow rate is further decreased, a backwater effect will drown the flow upstream.

Wave motion is that phase of flow in a gravitational field which has not only received the most attention over the greatest number of years but which has also succumbed most completely to mathematical analysis. Now waves in general are both unsteady and nonuniform, like those seen here, but we shall deal only with the types that can be made to appear steady by the principle of relative motion. This means that the wave profile must move relatively to the fluid itself with a constant speed, known as the celerity c , and hence without change in form. The celerity of a wave depends, in addition to gravity, upon three factors: the depth d of the fluid stratum over which it moves, the wave length λ , and the wave amplitude a . If the wave length is large compared to the depth, its effect is negligible and it is the depth that controls the celerity, as in the case of tides. If, on the contrary, the depth is large compared to the wave length, it is the wave length that controls the celerity, as in storm waves in the ocean. Increasing the amplitude of a wave causes its celerity to increase in either case; this effect is limited, however, for a wave crest curves more and more rapidly with increasing amplitude and eventually peaks and breaks. In any case, as seen from the paths of these suspended particles, the celerity of the profile and the locally induced velocity of the water are quite different.

Oscillatory waves are readily generated by the harmonic displacement of a wall. They can be of either the shallow-water or deep-water type, depending upon the ratio of wave length to depth. If a train of oscillatory waves is reflected by a stationary wall, as at the left, the result is a series of standing waves, the profiles of which have no longitudinal but only vertical motion; standing waves of great length are often formed in harbors. Generation of what is known as a solitary wave requires the displacement of the wall a finite distance in the positive or negative sense. The

resulting wave rapidly assumes a stable form and can be reflected
repeatedly between the end walls of the channel.

Surge generated
   by wall
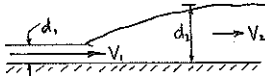
Undular surge
   generated by
   gate closure

Breaking surge

Very high surge

What is called a surge entails an abrupt change from one depth to
another and requires the continuous displacement of the generating
wall in one direction. The same phenomenon results from the abrupt
adjustment of a gate in a uniform flow. Surges in which the ampli-
tude is much less than the initial depth involve a train of waves
oscillating about the final depth; if the amplitude is nearly as
great as the depth, the first wave breaks and gives the front a
very turbulent form. As the relative amplitude continues to in-
crease, the celerity increases without limit.

Hydraulic jump

Animation

Although surges are regularly encountered in estuaries as tidal
bores which are sometimes more than 10 feet high, they are of even
greater engineering interest in their standing form called the hy-
draulic jump. This is an occurrence sometimes purposely brought
about by backwater downstream from a spillway or sluice gate to
change high-velocity to low-velocity flow for purposes of struc-
tural safety. Since the flow is essentially uniform before and
after the breaking front, the relationship between the celerity
and the change in depth can readily be evaluated through use of
the simplified continuity and impulse-momentum relationships for
one-dimensional flow, with this result. The jump is seen to take
place between the supercritical and subcritical regimes without
benefit of channel constriction. In fact, the equation for the
surge celerity, which now equals the velocity of the oncoming flow,
can be written in terms of the Froude number. Evidently $F = 1$
when $d_2 = d_1$. Through additional use of the work-energy rela-
tionship, it will be found that a considerable loss in total head
must take place if the other two equations are to be satisfied.
This results from the generation of turbulence at the breaking
front, the energy of which is rapidly dissipated through viscous
shear, as will be described in the next film of the series.

$$d_1 V_1 = d_1 V_1 = q_f$$
$$\frac{\gamma d_1^2}{2} - \frac{\gamma d_1^2}{2} = q_f \rho (V_1 - V_2)$$
$$c = V_1 = \sqrt{gd_1}\left[\frac{1}{2}\frac{d_2}{d_1}\left(\frac{d_2}{d_1} + 1\right)\right]^{1/2}$$
$$\frac{V_1}{\sqrt{gd_1}} = F_1 = \left[\frac{1}{2}\frac{d_2}{d_1}\left(\frac{d_2}{d_1} + 1\right)\right]^{1/2}$$

H line added to
   jump profile

Lecture

You have just seen examples of various flows in which gravity is
commonly considered to play an essential role. In the numerical
analysis of such problems, g, the fluid weight per unit mass, is
treated as a constant, because the flows presumably take place
under almost identical gravitational conditions. Moreover, the
free surface involved is usually that of water under air, the simul-
taneous motion of the air being ignored because of its much smaller
density. Now quite similar conditions would obtain between two
fluids of more nearly the same density - like cold water under warm
water, or cold air under warm air. For example, these two reser-
voirs hold water under air and salt water under fresh water. As
the combined unit is displaced, very much the same wave motion is
produced at both interfaces, except that the brine-water combina-
tion seems to follow a totally different time scale.

Close-up of flow

Board

$$g = \frac{\gamma}{\rho}$$

$$g' = \frac{\Delta\rho}{\rho}g = \frac{\Delta\gamma}{\rho}$$

$$0 < \frac{\rho_1}{\rho_2} < 1$$

What is actually different between the two flows is the effective acceleration of gravity, which varies in proportion to the relative difference in density - or, more significantly, is equal to the differential weight per unit mass. If due account is taken of the fact that both fluids are involved in the acceleration, much the same procedures can be used in the analysis of flows in which the ratio of fluid densities varies between the limits of nearly zero, for air and water, and nearly unity, for similar fluids of slightly different composition or temperature. In the remaining scenes of this film, layers of air, fresh water, and brine will be used to reproduce - apparently in slow motion - many of the occurrences previously shown. These phenomena are commonly found at natural scale in the atmosphere, and in the oceans, lakes, and rivers of the earth.

Fog at mountain pass

Spillway overflow

Underflow front

Passage of front
over rise

Reflection of
front by wall

Surface wave

Interfacial wave

The flow of cold, foggy air over the high point of a mountain pass into a warmer zone is modeled in a laboratory flume by the discharge of a silt-laden stratum from a reservoir. A fair reproduction of the hydraulic jump that normally forms below a spillway is seen downstream. The abrupt front of a gravity underflow typifies not only the so-called dambreak wave but also the saline tidal wedge in an estuary, or even a dust storm or an atmospheric cold front. Not only does it behave, at a constriction, like water under air, but it forms a surge travelling back upon itself when reflected by a barrier.

A wave at the free surface between water and air necessarily involves liquid strata of nearly equal density as though no density difference existed. A wave at the interface between two such strata, however, will affect them in reciprocal fashion without disturbing the upper surface. The wave celerity, obviously, is then greatly reduced. Wave resistance such as that encountered by ships will receive further attention in the final film of this series. For the present it should be noted that a ship will also be resisted by subsurface waves generated at the interface between layers of different temperature or salinity. As a final scene, there will be modeled, at reduced scale, waves that are formed in atmospheric layers of different temperature carried as wind over a mountain range. It might be remarked, in closing this laboratory simulation of natural gravitational effects, that not only is the atmosphere reproduced effectively by water and the thermal stratification by layers of different salinity, but - for reasons of experimental convenience - the apparently moving body of fluid is actually stationary and the schematic model is being towed through it - suspended, like the camera itself, upside down from a towing carriage at the level of the topmost - here seen as the bottommost - water layer.

Ship wave at
interface

Stratified flow over
schematic mountain